

New Scaling Law for Deuteron Production in Ultra-Relativistic Nucleus Nucleus Collisions

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Abstract

Deuteron production in S and Pb induced collisions at beam energies of 200 and 160 AGeV is studied in the framework of the transport theoretical approach RQMD. Strong transverse flow invalidates the differential coalescence formula in momentum space. The transverse momentum integrated d yields scale in a broad rapidity interval with the squared proton densities and inversely with the produced particle rapidity densities. This kind of scaling can be linked to constant relative sizes of nucleon and pion sources at freeze-out. With increasing projectile mass the nucleon source blows up stronger than the pion source. As a result, the scaled deuteron densities drop in central Pb+Pb collisions by 15 percent as compared to S induced reactions.

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1 Introduction

High energy nucleus-nucleus collisions have been explored for several years as a means of creating novel states of matter [1, 2]. Observables which are di-

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rectly related to the space-time structure of the reactions are of particular interest, because they allow for a final ‘snapshot’ of the hadron emitting source in Minkovski space. The cluster production rate depends on the distances between nucleons in the formation process. Since only nucleons with similar positions and momenta fit into the cluster wave function, the formation probability for a cluster with mass A is actually approximately proportional to the average of the $(A-1)$ -power of the nucleon phasespace density. The cluster production rates do not change once entropy production has ceased in the collision [3]. Hence, clusters are sensitive to the state of the system at much earlier times than when they are formed. In this paper, we suggest observables whose measurement will help unravel the relative sizes of proton and pion sources in AA collisions at energies above 100 AGeV. The final source sizes reflect a different (de-)coupling strength of particle species in the created medium [4, 5] and also differences in their production mechanisms [6]. Just after hadronization the baryon source at midrapidity might have a size different from the source of secondaries. For instance, the transverse size of the baryon source is expected to be relatively smaller in symmetric AA collisions, because projectile stopping is a function of the target thickness.
¹ Since nucleons have a larger cross section than pions in hadronic matter which contains only a small baryon fraction, they decouple from the medium at lower density or – equivalently – show a larger size at freeze-out. It would be interesting to know the dominating effects which determine the final size of the proton source as compared to the total source, mostly pions at large beam energy.

2 Scaling Laws

It was suggested a long time ago that the invariant yields of light nuclei with mass A scale with the A -th power of the proton yields

$$E_A \frac{d^3 N_A}{dp_A^3} = B_A \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A \quad (1)$$

¹ In the average a nucleon has to propagate through a lot of nuclear material – on the order of 12 fm at nuclear ground state density – to end up in the central rapidity region at CERN energy (200AGeV). The longitudinal velocity has to be degraded by three units of rapidity. A single inelastic NN collision shifts a nucleon only by about 0.7 units. The effective shift in pA collisions is even smaller, approximately 0.5 per collision [7].

with a universal scale factor B_A determined by the nuclear parameters of the ingoing projectile and target [8]. This scaling relation was very successful in describing the yields of light nuclei in nucleus-nucleus interactions at Bevalac and SIS energies around 1 AGeV [3]. Note that the scaling factor B_A has been chosen originally as volume-independent. This may indicate that the expansion at low energies proceeds practically isentropically. However, scaling is violated by a factor of up to 40 in nuclear collisions at the beam energy of 10-15 AGeV [9]. Production of additional particles which has been neglected in the scaling relation eq. 1 may impact cluster production at the higher beam energy. Since additional particles can interact with the original nucleons as well, the system must expand to a lower density before it can break up. In particular, entropy generating processes like the formation and decay of resonances play an important role in the space-time evolution of the heavy ion collisions according to calculations with the transport model RQMD [10, 11]. Recently, the spectra of deuterons and protons produced in Si(14.5AGeV) collisions with a heavy target which were measured by the groups E814 [9] and E802 [12] have been analysed and compared to calculations with RQMD [13]. The data were interpreted as evidence for considerable transverse expansion beyond the volume given by the geometric overlap region of the colliding nuclei. The transverse expansion is most pronounced for central collisions. Of course, this is expected if the interactions between original nucleons and secondary particles are of importance for cluster formation.

In this paper we suggest a new scaling relation for deuteron yields which may be appropriate for ultrarelativistic energies. We suggest that the d yields scale inversely with the produced particle rapidity density in AA collisions at sufficiently large beam energies. We express the parameter B_d as

$$B_d = c \cdot (dN^-/dy)^{-1} \quad , \quad (2)$$

which may reflect better the conditions of nuclear interactions at ultrarelativistic energies than B_A definitions ignoring particle production. The so defined B_d contains implicitly a dependence on the source size of the reaction, because the particle rapidity density ² is proportional to the freeze-out

² We have chosen – somewhat arbitrarily – the negative particles for the scaling law. Note that mesons are populating the three charge states in approximately equal amounts at ultrarelativistic energies. We have included feed-down from weak decays (except Λ and K_S) in the negatives, because it is experimentally much easier to subtract hyperon

volume, if hadrons freeze-out at constant density. The radius parameters of Bose-Einstein correlations scale rather well with the rapidity density as $\sim (dN^{+/-}/dy)^{1/3}$. This dependence was reported for high energy hh and AA reactions by various experimental groups (AFS, UA1, E802, NA35 and NA44) and is consistent with freeze-out at constant density. (See ref. [14] for a review on recent HBT results.) One could infer from a B_d dependence on the produced particle rapidity density as in eq. 2 that the nucleon source size scales with the produced particle source. By using eq. 2 we hope to incorporate the most important aspect of particle production on nuclear coalescence probabilities. The formula eq. 2 should be applied only for collisions which are dominated by particle production, i.e. with a ratio N_π/N_B much larger than 1. Therefore we test the scaling relation for A on A reactions which are currently under experimental study at the CERN-SPS (160-200AGeV). Here we are using phasespace distributions calculated with the RQMD model (version 1.09) [6, 15, 16] in combination with the Wigner function method for cluster coalescence [17, 18, 19]. There are only preliminary experimental data on cluster and anti-cluster production in AA collisions at these energies available from the NA44 and NA52 groups [20, 21]. Comparisons with RQMD calculations are under way [22, 23].

RQMD is a multiple-collision approach in which the secondary particles emerge as fragmentation products of decaying color strings, ropes and resonances. Afterwards they may interact with each other and the ingoing nucleons. Nuclear cluster formation is not included dynamically in RQMD. It is added after the strong interactions have ceased and the particles are streaming freely. The deuteron yields are calculated from the product of proton and neutron source functions (g_p and g_n) at freeze-out and the coalescence factor, integrated over phasespace:

$$N_d = \int d^4x_1 d^4x_2 \int dp_1 dp_2 g_p(x_1, \vec{p}_1) g_n(x_2, \vec{p}_2) p_d(1, 2) \quad . \quad (3)$$

The coalescence factor p_d is taken as

$$p_d = 3/8 \cdot f_W^d(\vec{x}_{CMS}, \vec{p}_{CMS}) \quad . \quad (4)$$

f_W^d denotes the Wigner transform of the deuteron wave function, with center-of-mass motion removed. Its arguments are the distance between the nucleons at the larger of the two freeze-out times and the relative momentum, all

feed-down from measured proton yields than to control the amount of meson decays, e.g. of η and η' . Our conclusions in this paper are not affected by this procedure.

values evaluated in the deuteron center of mass frame where $\vec{p}_1 + \vec{p}_2 = \vec{0}$. Here we use a harmonic oscillator wave function with parameters as employed for d coalescence calculations in [18]. The prefactor in eq. 4 comes from statistical isospin and spin projection of a np pair onto the corresponding deuteron quantum numbers. In the semiclassical RQMD model the final particle energies are set on-shell. Thus it is implicitly assumed by applying eq. 3 that the true quantum-mechanical source functions at freeze-out have negligible energy dependence on the scale of the deuteron binding energy around the mass shell.

The calculated rapidity distributions of deuterons and protons (without feed-down from weak decays) are displayed in fig. 1. The RQMD calculations have been carried out for three different systems, S on S (impact parameter $b < 1$ fm), S on W ($b < 4$ fm), both at a projectile energy of 200 AGeV, and Pb on Pb ($b < 1$ fm), with beam energy 160 AGeV. The different shapes of the baryon distributions reflect the dependence of the stopping power on target and projectile mass number and are discussed in [15]. The calculated d/p ratios are less than one percent in the central rapidity region, clearly smaller (by an order of magnitude) than the data measured at the lower beam energies of 10-15 AGeV [12].

A main motivation to compare calculated deuteron yields to scaling relations like eqs. 1,2 is that some basic aspect of the dynamics – more particles need a larger freeze-out volume – is accounted for. Other collective dynamical effects in ultrarelativistic collisions like collective flow may lead to scaling violations. Indeed, an immediate consequence of eq. 1 is that the transverse mass spectrum of the nucleon clusters will have the same slope as the primordial nucleon spectra. This results from eq. 3 only, if the freeze-out distribution g_N factorizes into a product of separate functions of position and momentum, which precludes flow. Transverse mass spectra calculated from RQMD are shown in fig. 2 (histograms). They are compared to fits with a Boltzmann distribution (symbols). The primordial proton spectra are well described by the thermal distributions with ‘apparent temperature’ (slope parameter) 180 MeV for S+S and 232 MeV for Pb+Pb collisions. At large m_t values the deuteron distribution is fitted by the Boltzmann distribution with same slope parameter as for the protons only in the S+S case. The slope parameter which is needed for the tail of the deuteron m_t distribution from Pb+Pb collisions has quite a different value, 300 MeV. Furthermore, the calculated deuteron distributions differ markedly from a thermal shape

if the spectra at low m_t values are taken into account. They exhibit a convex curvature (shoulder-arm shape) which is characteristic for the presence of transverse flow. We may therefore conclude that the coalescence formula eq. 1 is in contradiction with the calculated momentum distributions and therefore useless for further consideration here.

However, we can consider the validity of a coalescence formula similar to eq. 1 in a modified form, for the particle yields after integration over transverse momentum

$$dN_d/dy = B_d \cdot (dN_p/dy)^2 \quad . \quad (5)$$

A B_d dependence which is inverse to the volume $V = \int d^3\sigma_\mu u^\mu(x)$ of the baryon source after freeze-out can be derived from coalescence formula eq. 3 even in the presence of flow. The velocity field $\vec{u}(x)$ acquires nonzero components in this case. (The 4-vector $d^3\sigma_\mu$ has only a nonzero value for the 0-th component equal to d^3x in the local rest system defined by $\vec{u}'(x) = \vec{0}$.) The derivation holds under fairly general conditions: not too large spreading of emission times, local statistical equilibrium, constant local baryon density and freeze-out temperature, and small flow gradients on the scale of the deuteron size. The last assumption is needed for a factorization of g_N to be valid locally. Assuming local statistical equilibrium at some freeze-out temperature the pion yield is proportional to its source volume as well. It is straightforward under these assumptions to calculate the relation between coalescence parameter B_d and produced particle yields for a hadron gas in equilibrium. We will not do this here, but instead we will discuss to which extent scaling holds according to the RQMD results.

The scaled d rapidity density $[dN_d/dy]/(dN_p/dy)^2 \cdot dN^-/dy$ will be constant if eqs. 2,5 are satisfied. This ratio – calculated from the RQMD results – is displayed in fig. 3 as a function of rapidity for the three colliding systems under consideration. Indeed, the scaled d rapidity density turns out to be remarkably independent of rapidity in a broad interval of about three units. (It is quite natural that scaling breaks down in the projectile and target fragmentation region, because the basic assumptions, in particular meson dominance, are not fulfilled here.) There is essentially no target dependence in S induced reactions by going from a S to a W target. While collisions with central impact parameters ($b < 1$ fm) were taken for the symmetric systems, the S+W events were generated in an impact parameter range up to 4 fm. We have calculated the d rapidity densities for S+W in different event classes defined

by centrality ($b < 1$ fm, $1 \text{ fm} < b < 2$ fm, and so on). We find that the scaled d distributions do not depend on b within this impact parameter range. While scaling holds separately for the Pb+Pb collisions, it becomes clear from fig. 3 that the scaled d yields are smaller in central Pb+Pb collisions – by about 15 % – than in the S+A reactions. This points towards a relative increase of the nucleon source in comparison to the produced particle source.

The relative increase of the nucleon source in central Pb+Pb collisions can be checked directly by looking at the RQMD freeze-out distributions (see fig. 4). Fig. 4 shows transverse freeze-out distributions for pions, protons and kaons at midrapidity ($y_{mid}-1 < y < y_{mid}+1$). The relative size of the proton source grows relatively to the meson source by going from S+S to Pb+Pb. While the ratio of proton to pion transverse RMS radius (at midrapidity) is 1.05 in S+S reactions (5.4 fm:5.1 fm), it increases to 1.19 in central Pb+Pb collisions (10.3 fm:8.6 fm). The system has expanded considerably before freeze-out, because the initial transverse RMS sizes of S (Pb) have values 2.4 (4.5) fm only. RQMD shows no indications for a narrower transverse source size of midrapidity protons, just the opposite. The RQMD freeze-out distributions are not consistent with scaling in any power of number of participants, produced particle or nucleon rapidity density. While the source distributions have approximately a Gaussian shape in S+S (and S+W), the transverse flow is clearly visible in the distributions for Pb+Pb. A blast wave has transported much of the material into the outer space leaving a hole in the interior region. Inspection of the reaction history in RQMD reveals why this effect depends – to some extent – on the absolute size of the reaction region. Starting at a value of zero, the transverse baryon flow develops initially more slowly than the pion flow. The responsible factor here is the mass difference between baryons and mesons which is not compensated by sufficiently strong interaction to form really one fluid. The time averaged flow of particles freezing-out is affected by the same mechanism. The different strength of the transverse flow is weighted more heavily in smaller systems even if the particle density was initially the same, because the meson source has evaporated and the interactions with baryons have ceased earlier than in central collisions of heavy ions like Pb+Pb.

3 Conclusions

We have studied deuteron production based on RQMD calculations for S and Pb induced collisions at beam energies of 200 and 160 AGeV. We find violation of the usual differential coalescence formula in momentum space, in particular in central Pb on Pb collisions, which is caused by the presence of strong transverse flow. Transverse momentum integrated d yields scale in a broad rapidity interval with the squared proton densities and inversely with the produced particle rapidity densities. We have argued that this kind of scaling can be linked to constant relative sizes of nucleon and pion source at freeze-out. With increasing the projectile mass, we find a blow-up of the nucleon source which is stronger than for the pion source. Therefore the scaled deuteron densities drop in central Pb+Pb collisions by 15 percent as compared to S induced reactions. The calculations which have been presented here demonstrate the usefulness of cluster measurements to extract information on the created source – particularly its size and the strength of flow – in ultrarelativistic nucleus-nucleus collisions.

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References

- [1] **Quark Matter '95**, Proceedings of the 11th international conference on Quark Matter, Monterey, U.S.A., 1995. A. Poskanzer et al., eds.: *Nucl. Phys. A* (1995) in print.
- [2] **Quark Matter '93**, Proceedings of the 10th international conference on Quark Matter, Borlange, Sweden, 1993. H-A. Gustafsson et al., eds.: *Nucl. Phys. A566* (1994).
- [3] L.P. Csernai and J.I. Kapusta: *Phys. Rep.* **131** (1986) 223.
- [4] K. Haglin and S. Pratt: *Phys. Lett.* **B328** (1994) 255.

- [5] J. Cleymans, K. Redlich, H. Satz, and E. Suhonen: *Z. f. Phys.* **C58** (1993) 347.
- [6] H. Sorge: *Phys. Lett.* **B344** (1995) 35.
- [7] W. Busza, R. Ledoux: *Ann. Rev. Nucl. Part. Sci.* **38** (1988) 119.
- [8] S.T. Butler and C.A. Pearson, *Phys. Rev.* **129** (1963) 836.
- [9] E814 Collaboration, J. Barrette et al., *Phys. Rev.* **C50** (1994) 1077.
- [10] H. Sorge, R. Mattiello, H. Stöcker, and W. Greiner: *Phys. Lett.* **B271** (1991) 37.
- [11] M. Hoffmann, R. Mattiello, H. Sorge, H. Stöcker, and W. Greiner: preprint UFTP-374 (1994), *Phys. Rev.* **C** (1995) in print.
- [12] E802 Collaboration, T. Abbott et al., *Phys. Rev.* **C50** (1994) 1024.
- [13] J. Nagle et al.: *Phys. Rev. Lett.* **73** (1994) 1219.
- [14] R. Stock: *Annalen der Physik* **48** (1991) 195.
- [15] H. Sorge: UFTP 376-94 (preprint), *Z. f. Phys.* **C** (1995) in print.
- [16] M. Berenguer, H. Sorge, and W. Greiner: *Phys. Lett.* **B332** (1994) 15.
- [17] E.A. Remler: *Ann. Phys. (NY)* **136** (1981) 293 and refs. therein.
- [18] M. Gyulassy, K. Frankel, and E.A. Remler: *Nucl. Phys.* **A402** (1983) 596.
- [19] R. Mattiello, A. Jahns, H. Stöcker, W. Greiner, and H. Sorge: *Phys. Rev. Lett.* **74** (1995) 2180.
- [20] J. Simon-Gillo et al., NA44 Collaboration: LA-UR-95-834 (preprint), Contribution to ‘**Quark Matter 95**’, in ref. [1].
- [21] F. Dittus et al., NA52 Collaboration: Contribution to ‘**Quark Matter 95**’, in ref. [1].
- [22] J. Nagle, S. Kumar, H. Sorge, and R. Mattiello: in Bulletin of the American Physical Society, (1995), and to be published.
- [23] J. Simon-Gillo: private communication.

Figure Captions:

Figure 1:

Final rapidity distributions of negatively charged hadrons, primordial protons and deuterons calculated from RQMD (version 1.09). The RQMD calculations have been carried out for three different systems, S on S (impact parameter $b < 1$ fm), S on W ($b < 4$ fm), both at a projectile energy of 200 AGeV, and Pb on Pb ($b < 1$ fm), with beam energy 160 AGeV. The distributions are displayed as histograms, straight line for protons, dashed for deuterons (multiplied with 50) and dotted for negatives (multiplied with 0.1). The rapidity is calculated in the equal-speed-system of projectile and target. The generated events for the symmetric systems have been reflected $z \rightarrow -z$ in order to improve statistics.

Figure 2:

Transverse mass spectra of primordial protons and deuterons in the rapidity window $y_{mid} - 0.5 < y < y_{mid} + 0.5$. The spectra have been calculated for S+S and Pb+Pb using the RQMD model under the same conditions as explained in caption to fig. 1. They are represented as histograms, protons from Pb+Pb (straight line), deuterons from Pb+Pb (dashed line), protons from S+S (dotted line), and deuterons from S+S (dashed-dotted line). The calculated spectra are compared to Boltzmann distributions $\sim m_t \cdot \exp(-m_t/T)$ with slope parameters fitted to the large m_t tail of the distributions, with T parameters (from top to bottom) 232, 180, 300 and 180 MeV.

Figure 3:

The scaled d rapidity density $[dN_d/dy/(dN_p/dy)^2] \cdot dN^-/dy$ as a function of rapidity. The RQMD calculations have been carried out for the same three systems as in fig. 1, S on S (straight line), S on W (dashed histogram) and Pb on Pb (dotted histogram).

Figure 4:

Distribution of transverse distances (to the collision center) $1/r_t dN/dr_t$ at freeze-out for different particle species in S on S (top), S on W (middle) and Pb on Pb (bottom) collisions. Only particles around central rapidity ($y_{mid} \pm 1$) are taken into account, primordial protons (straight line), pions

(dashed histogram), and neutral (anti-)kaons $K^0 + \bar{K}^0$ (dotted histogram). Note that the pion and kaon distributions have been renormalized that the integral gives the same yield as for the protons.

Figure 1:

Figure 2:

Figure 3:

Figure 4:

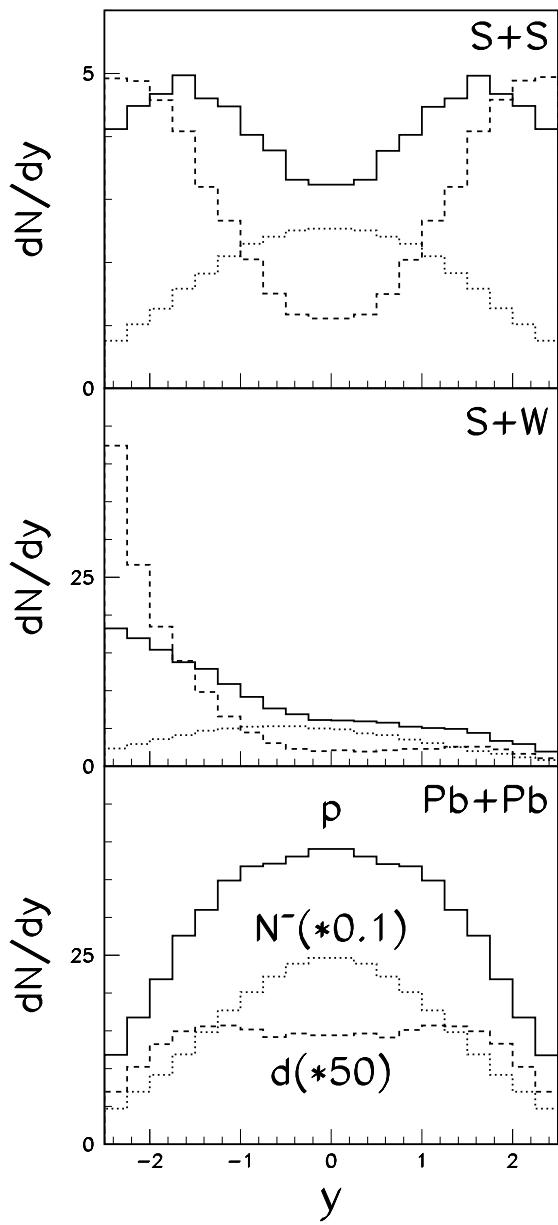


Figure 1:

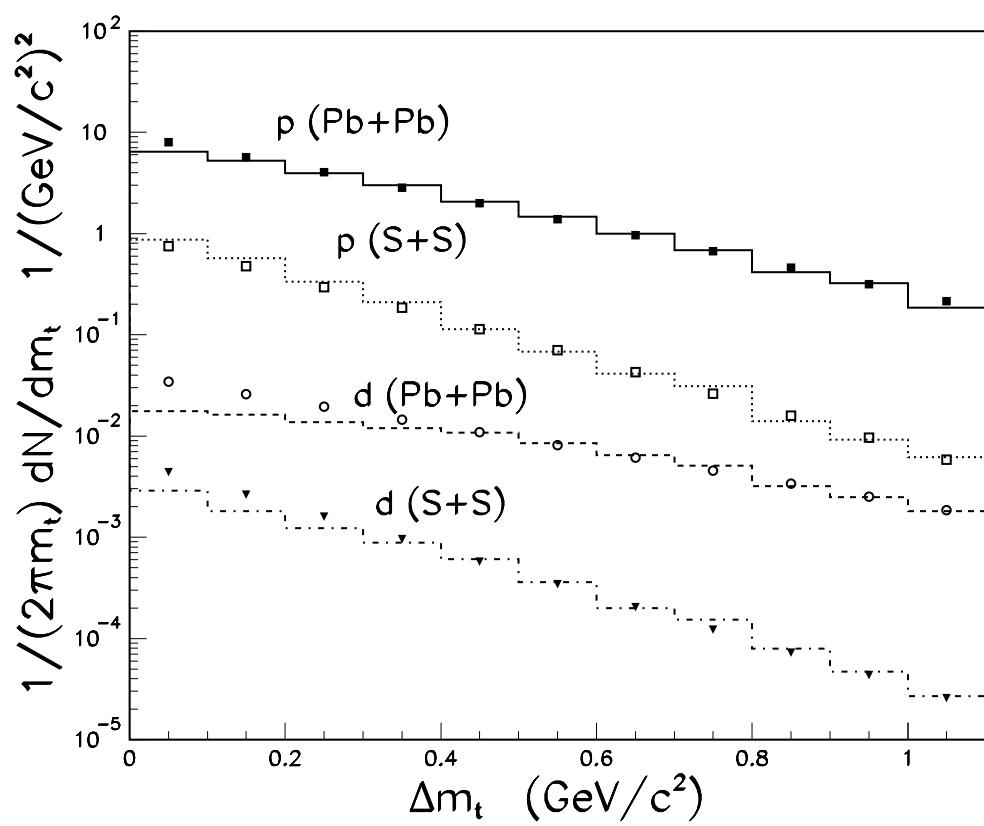


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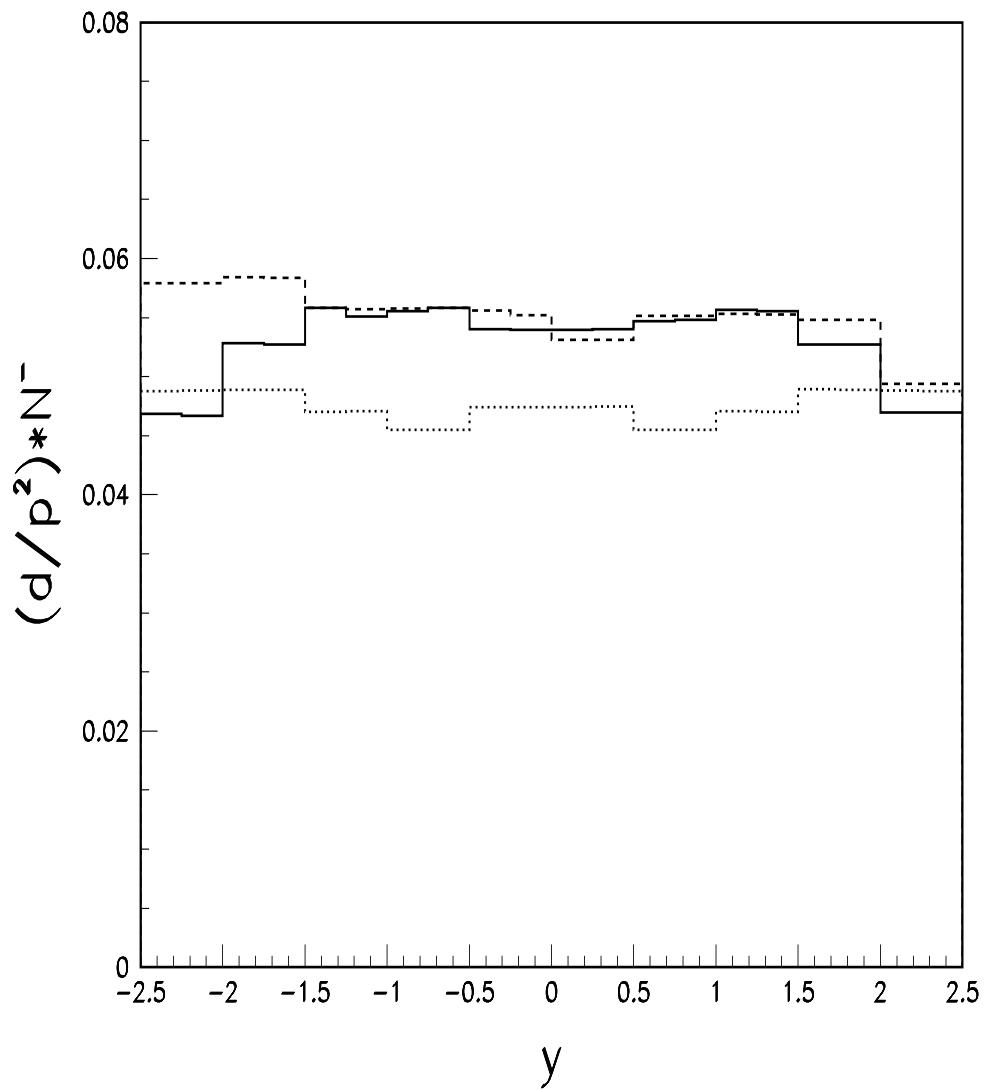


Figure 3:
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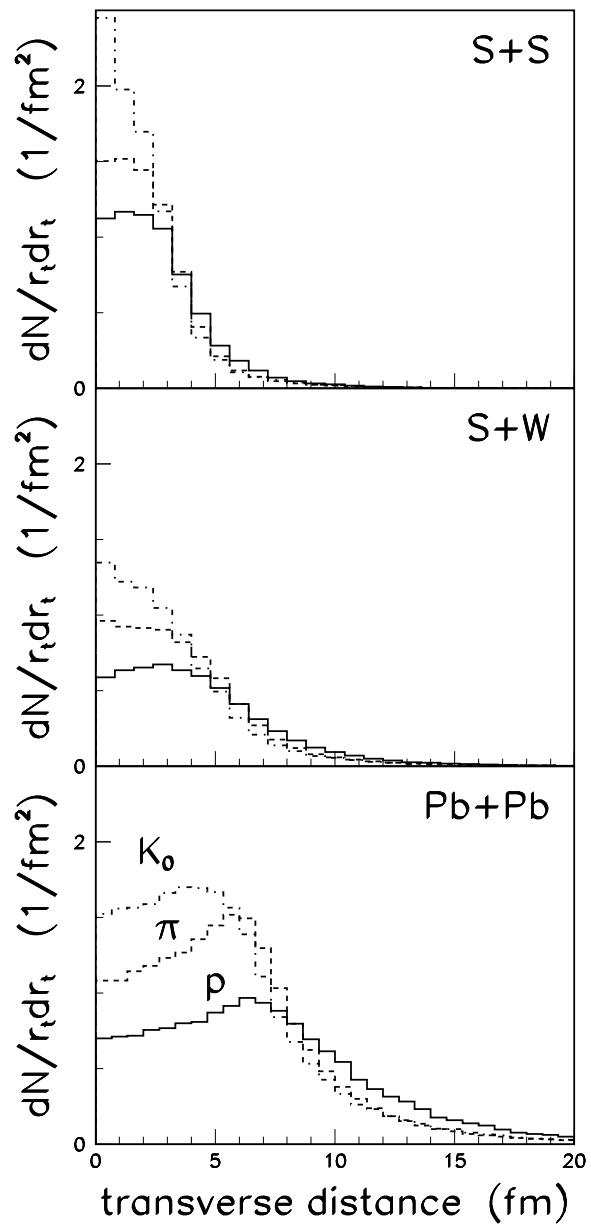


Figure 4: